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Boundary element analysis of the seismic response of gravity dams

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ABSTRACT

A boundary element technique for the seismic response analysis of gravity dam-reservoir-foundation system is presented. Analysis is carried out in the frequency domain and a substructure approach is used. The reservoir substructure is divided into two parts: a near field of specified irregular geometry modeled by boundary elements and an infinite far field of uniform cross section modeled by 1D finite elements. The energy absorption along the reservoir bed is modeled by an approximate wave propogation model. The dam is discretized by finite elements. The foundation soil, idealized as an elastic half plane, is modeled by boundary elements. To enhance the computational efficiency of the procedure, a frequency independent fundamental solution is adopted for the reservoir, and the equations of the motion are transformed to the first few modal coordinates of the dam-foundation system. The complex frequency response functions of Pine Flat dam are obtained for various conditions of reservoir and foundation. A response time history analysis is carried out for the Taft ground motion.

INTRODUCTION

Analytical procedures for determining the seismic response of gravity dams are now fairly well developed. Since the response of the dam is affected both by the reservoir induced hydrodynamic forces and interaction with the flexible foundation, a substructuring technique is most effective in the analysis. The dam and the reservoir substructures are usually represented by finite elements, while the foundation is treated as elastic or viscoelastic halfspace. A finite element representation of the foundation is also possible provided a rigid boundary exists or an artificial boundary that prevents the reflection of outgoing waves can be introduced.

The reservoir model must account for the energy lost in waves radiating towards infinity and through the absorption effects along the reservoir bottom. To model the infinite radiation, the reservoir is divided into two parts: a near field of specified irregular geometry and a far field of uniform cross-section. The latter is usually modeled by 1D finite elements (Hall and Chopra 1980). The energy dissipation through partial wave absorption in a flexible reservoir bottom can be accounted for by using a 1D wave propogation model proposed by Hall and Chopra (1980).

In recent years boundary element method (BEM) has emerged as an effective alternative to

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the finite element method (FEM) for the seismic analysis of gravity dams. Analytical procedures based on BEM have been developed for reservoir vibration by Hanna and Humar (1982), and Humar and Jablonski (1988). In a recent study, Medina et al. (1990) have used the BEM for the response analysis of the complete dam-reservoir-foundation system. However, their study extends only to the computation of frequency response functions; and analytical solutions for real earthquake motions have not been obtained.

The present study extends the BEM procedures to the response analysis for a realistic earthquake motion using a Fourier synthesis approach. In the analysis for a real ground motion of random nature, it is important to pay attention to the computational efficiency of the procedure. The traditional BEM uses a frequency dependent fundamental solution, implying that the BEM matrices must be evaluated for each excitation frequency, a computational task of considerable magnitude. The present study uses a frequency independent fundamental solution (Nardini and Brebbia 1984; Tsai 1987) for the near field of the reservoir, the far field being modeled by 1D finite elements. To minimize the computations, the reservoir bottom absorption is represented by 1D wave propogation model and the far field of the reservoir is assumed to possess rigid bottom. The dam is discretized by finite elements. The foundation soil is treated as a half plane, modeled by boundary elements along the foundation surface (Abdalla 1984). The effect of cross coupling between the foundation portion below the reservoir and that below the dam has been found to be small by other researchers (Fenves and Chopra 1984) and is therefore ignored. The analysis is carried out in the frequency domain using Fast Fourier Transform (FFT).

Brief formulations of the substructure models are provided in the following section. The boundary element procedure is then applied to the analysis of Pine Flat dam, for its response to Taft ground motion.

ANALYTICAL FORMULATIONS

Reservoir Hydrodynamic Force

The 2D wave equation governing the reservoir fluid vibration (Fig. 1) is given by

$$\nabla^2 p + k^2 p = 0 \tag{1}$$

where p is the hydrodynamic pressure, k the wave number $= \Omega/c$, Ω the excitation frequency, and c the wave velocity in water. By using the weighted residual technique, with the weighting function p_i^* chosen to be equal to the static solution for a unit source at point i, Eq. 1 can be transformed into an integral equation involving integrals on the boundary and the domain integral $k^2 \int \nabla^2 p . p_i^* dA$. This domain integral can also be transformed to boundary integrals by assuming the following particular solution for p

$$p(x, y, \Omega) = \sum_{m=1}^{M} \alpha_m(\Omega) f_m(x, y) \tag{2}$$

in which $\alpha_m(\Omega)$ is an unknown coefficient, and $f_m(x,y) = \nabla^2 \psi^m(x,y)$ is a suitably chosen harmonic function. By discretizing the near field boundary into a series of segments termed boundary elements, with the pressure and pressure gradients as well as ψ and its derivative $\eta = \partial\psi/\partial n$ assumed to vary in a prescribed manner over each element, the boundary integral equations, can be represented in a discretized form as

$$\mathbf{Hp} - \mathbf{Gq} = k^2 \left(-\mathbf{H}\psi + \mathbf{G}\eta\right) \alpha \tag{3}$$

where $\mathbf{H}$ and $\mathbf{G}$ are matrices of the boundary integrals involving $q_i^* = \partial p_i^*/\partial n$ and p_i^* respectively, $\mathbf{p}$ is the vector of pressures and $\mathbf{q}$ the vector of pressure gradients.

In the present formulation, p_i^* is given by

$$p_i^* = -(1/2\pi) ln \left(r_{ij}/r'_{ij}\right) \tag{4}$$

and function f is chosen as

$$f_j(x_i, y_i) = r'_{ij} - r_{ij} \tag{5}$$

where r_{ij} and r'_{ij} are the distances to the field point j respectively from the source point and the mirror image of the source point. The solution p_i^* automatically satisfies the zero pressure condition on the surface of the reservoir.

The vector α can be expressed in terms of $\mathbf{p}$ and a matrix $\mathbf{F}$ containing the values of f_m at the field points. Substitution in Eq. 3 gives

$$\left(\mathbf{H} - k^2\tilde{\mathbf{M}}\right)\mathbf{p} = \mathbf{G}\mathbf{q} \tag{6}$$

where $\tilde{\mathbf{M}} = (\mathbf{G}\eta - \mathbf{H}\psi)\,\mathbf{F}^{-1}$

The respective boundary conditions along the dam face and reservoir bottom are

$$q(s, \Omega) = -\rho_w a_n(s) \tag{7a}$$

$$q(s', \Omega) = -\rho_w\, b_n(s') - \iota\Omega\beta p(s', \Omega) \tag{7b}$$

where ρ_w is the mass density of water; a_n the acceleration of the dam face along the normal; β the wave absorption parameter $= \rho_w/\rho_r c_r$; ρ_r and c_r the mass density and the compression wave velocity of the reservoir bottom; b_n the free field acceleration of reservoir bottom along the normal; and s' the coordinate along reservoir bottom. It is perhaps more meaningful to represent the wave absorption along the reservoir bottom through a wave reflection parameter $\tilde{\alpha}$ given by $\tilde{\alpha} = (1 - \beta c)/(1 + \beta c)$. A value of $\tilde{\alpha} = 1.0$ represents a nonabsorptive reservoir bottom.

Along the interface with far field, vectors $\mathbf{p}$ and $\mathbf{q}$ are related by (Hall and Chopra 1980)

$$\mathbf{p} = \mathbf{\Gamma}\, \upsilon + \rho_w\, \mathbf{\Gamma}\, \mathbf{K}^{-2}\, \mathbf{\Gamma}^T\, \mathbf{d} \tag{8a}$$

$$\mathbf{q} = -\mathbf{\Gamma}\, \mathbf{K}\, \upsilon \tag{8b}$$

where γ_n and λ_n are respectively the eigenvectors and eigenvalues of the far field; $\mathbf{\Gamma}$ the matrix of eigenvectors; υ is a vector of modal coordinates; $\mathbf{K}$ a diagonal matrix with elements $\kappa_1, \kappa_2, \cdots, \kappa_L$; L the number of finite element nodes on the interface boundary; $\kappa_n = \sqrt{\lambda_n - k^2}$ and $\mathbf{d}$ a vector with only one non-zero entry equal to an input acceleration a_y corresponding to the base node.

Foundation Impedance Matrix

The boundary integral equations for an homogeneous, isotropic and linear elastic soil region is given by

$$\beta^P u_j(P) + \int_S T_{ji}^*(P,Q) u_i(Q)\, ds - \int_S t_i(Q)\, U_{ji}^*(P,Q) ds = 0 \tag{9}$$

where P is the source point and Q the field point; $\beta^P = 0.5$ for P on a smooth boundary; u_i and t_i denote displacement and traction vectors; U^*_{ji} and T^*_{ji} are the fundamental solutions (Abdalla 1984); S is the boundary of the soil domain and indicial notations are used. Since the kernels in the integrals decay rapidly with distance, only the dam-foundation interface along with a finite portion of the soil surface needs to be discretized. The infinite radiation is automatically accounted for by the fundamental solution.

The complex valued impedance matrix of the foundation, required in the substructure analysis, is obtained by first evaluating the flexibility matrix corresponding to the degrees of freedom of the dam-foundation interface nodes and then taking its inverse. For a horizontal surface, with equally spaced interface nodes, the complete flexibility matrix can be constructed from 2 sets of analyses, one for a unit harmonic horizontal traction and other for a unit harmonic vertical traction applied at any one node.

Coupled Dam-Reservoir-Foundation System

The equation of motion of the dam in frequency domain is given by

$$\left\{-\Omega^2\mathbf{M} + (1+\iota\eta_d)\mathbf{K}\right\} \mathbf{U}^l(\Omega) = -\mathbf{E}^l + \mathbf{R}^l(\Omega) \tag{10}$$

where $\mathbf{M}, \mathbf{K}$ represent respectively the mass and the stiffness matrices of the dam substructure including the dam-foundation interface nodes; η_d is the hysteretic damping factor; $\{\mathbf{U}^l(\Omega)\}$ is the nodal displacement vector relative to the free field; $l = x, y$ is the direction of excitation; $\{\mathbf{E}^x\}^T = \{m_1\ 0\ m_2\ 0\ .....\ m_N\ 0\}$ and $\{\mathbf{E}^y\}^T = \{0\ m_1\ 0\ m_2\ .....\ 0\ m_N\}$; $\{\mathbf{R}^l(\Omega)\}$ is the nodal load vector, having non-zero quantities corresponding to the upstream face and the base DOF of the dam. The nodal load vector is composed of two vectors: $\mathbf{R}^l_h$ denoting the hydrodynamic forces along the upstream face of the dam, and $\mathbf{R}^l_b$ the forces along the dam-soil interface nodes. The latter are given by

$$\mathbf{R}^l_b(\Omega) = -\mathbf{S}_f(\Omega)\,\mathbf{U}^l_b(\Omega) \tag{11}$$

where $\mathbf{S}_f(\Omega)$ is complex valued soil impedance matrix.

The size of the problem represented by Eq. 10 can be reduced by using the Rayleigh Ritz method, in which the first Nm undamped mode shapes of the dam-foundation system are used as Ritz vectors. The transformed equations become

$$\left\{-\Omega^2\mathbf{I} + (1+\iota\eta_d)\mathbf{\Lambda} + \mathbf{\Phi}^T\tilde{\mathbf{S}}_{0f}(\Omega)\,\mathbf{\Phi}\right\} \mathbf{Z}^l(\Omega) = -\mathbf{\Phi}^T\mathbf{E}^l + \mathbf{\Phi}^T\mathbf{R}^l_h(\Omega) \tag{12}$$

where

$$\tilde{\mathbf{S}}_{0f}(\Omega) = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{S}_f(\Omega) - (1+\iota\eta_d)\mathbf{S}^r_f(\Omega_0) \end{pmatrix}$$

$\mathbf{\Lambda}$ is the diagonal matrix of the squared frequencies ω_j^2, $Z^l_j(\Omega)$ is the jth modal coordinate; ω_j and ϕ_j are the natural frequencies and mode shapes; $\mathbf{S}^r_f(\Omega_0)$ is the real part of the soil impedance matrix corresponding to a small frequency value Ω_0.

The hydrodynamic force vector $\mathbf{R}^l_h$ can be expressed as

$$\mathbf{R}^l_h(\Omega) = \mathbf{R}^l_o(\Omega) - \Omega^2 \sum_{j=1}^{Nm} Z^l_j(\Omega)\mathbf{R}^f_j(\Omega) \tag{12}$$

where $\mathbf{R}_o^l(\Omega)$, $\mathbf{R}_j^f(\Omega)$ are the nodal forces along dam's upstream face equivalent to the hydrodynamic pressures $p_o^l(s,\Omega)$ and $p_j^f(s,\Omega)$ respectively. Pressures $p_o^l(s,\Omega)$ are obtained from boundary conditions corresponding to a rigid dam and free field earthquake motion, while the pressures $p_j^f(s,\Omega)$ are obtained with the motion of the upstream face of the dam being equal to that in the jth mode.

EARTHQUAKE ANALYSIS OF PINE FLAT DAM

To illustrate the analytical procedure developed here, the seismic response of a non-overflow section of Pine Flat Dam, California is analysed for Taft ground motion. The cross section dimensions and the finite element mesh for the analysis of this dam are identical to those given in Fenves and Chopra (1984). For the dam material, the Young's modulus of elasticity $E_d = 3.25 \times 10^6$ psi, unit weight=155 lb/ft^3, Poisson's ratio = 0.2, and the hysteretic damping factor $\eta_d = 0.1$. The foundation is idealized as a homogeneous, isotropic, elastic half-plane. The properties of the foundation material are $E_f = 3.25 \times 10^6$ psi, so that the elastic modulii ratio $E_f/E_d = 1$, unit weight=165 lb/ft^3, and Poisson's ratio = 1/3. The full reservoir has a constant depth of 381 ft. The pressure wave velocity is 4720 ft/sec, and the unit weight of water is 62.4 lb/ft^3. Two values of wave reflection coefficient $\tilde{\alpha} = 1.0$ and $\tilde{\alpha} = 0.75$ are considered. The length of the near field L_f is taken as 0.1H, for the case of fully rigid reservoir bottom and as 2.5H for the case of absorptive bottom. The dam-foundation interface along with a length of 400 ft on either side of the dam base is discretized for the BEM analysis of the foundation.

Taft ground motion of July 1952 earthquake is chosen as free-field input, its S69E and vertical components acting along the transverse and vertical direction of the dam respectively. Using the analytical procedure discussed, the seismic response analysis of the dam section to the Taft motion is carried out considering all the significant interaction components. The analysis is performed by adopting the first 5 mode shapes of the dam, for a rigid foundation case, and 10 mode shapes of the associated dam-foundation system, when the foundation is flexible.

The complex frequency response functions for the absolute horizontal crest acceleration and the time history of the horizontal crest displacements for different conditions of foundation, reservoir water, bottom absorption and excitation direction are presented. For the frequency domain analysis, 2048 time steps of 0.02 s are used. The first 20 s of the excitation is taken as equal to the ground acceleration values for the Taft motion, the remaining period is made up of a grace band of zeros to avoid the aliasing errors inherent in discrete Fourier transforms.

The complex frequency response functions for the crest acceleration due to horizontal excitation are shown in Fig. 2. Reservoir bottom absorption reduces the peak response near the fundamental frequency for both rigid and flexible foundation, although the effect is not as pronounced in the latter case. The absorption effects are less significant at higher frequencies. Flexibility of foundation causes a reduction in the crest acceleration. The frequency response functions due to vertical excitation are shown in Fig. 3. The response exhibits infinite peaks for a non-absorptive bottom, whether or not the foundation is flexibile. Wave absorption at reservoir bottom results in a drastic reduction in the peak response. As for a horizontal excitation, foundation flexibility causes a reduction in the overall crest acceleration.

Figure 4 presents the first 15 s of time history of the horizontal crest displacement of Pine Flat dam subjected to Taft ground motions. It is apparent from these results that the responses to the S69E (transverse) component of Taft motion are quite similar for $\tilde{\alpha} = 1.0$ (non-absorptive) and 0.75 (absorptive reservoir bottom), irrespective of the foundation flexibility. This implies that the response due to the transverse ground motion is less sensitive to the reservoir bottom absorption. For the case of vertical excitation of the system with non-absorptive bottom, the peaks in the complex frequency response function render the results of an FFT analysis invalid because of aliasing. Hence, only results corresponding to absorptive bottom ($\tilde{\alpha} = 0.75$) are presented. The overall effect of foundation flexibility is an increase in crest displacement, both for horizontal and a vertical excitation.

For vertical excitation, the approximate 1D wave absorption model (Hall and Chopra 1980) has been found to compare well with a more rigorous model studied by Medina et al. (1990). For horizontal excitation, Medina has cited that the approximate model underestimates the complex frequency response. However, since the reservoir bottom absorption plays only a minor role in the response of a dam to realistic horizontal excitation, the approximation involved in the 1D model has very little effect on the final response.

CONCLUSIONS

The boundary element procedure developed in the present study is applicable to the seismic response analysis of a general gravity dam-reservoir-foundation system of arbitrary geometry. The computational efficiency of the procedure is enhanced by the use of a frequency independent model of the reservoir, and the tranformation of the equations of motion to modal coordinate of a dam-foundation system. The analytical results show that the effect of reservoir bottom absorption is not very significant for the response of the dam to a horizontal excitation, but is important in the response to a vertical excitation. In either case, the approximate 1D wave propogation model yields reasonably accurate results and a more rigorous approach for modeling the reservoir bottom may not be necessary. On the other hand, use of approximate model results in considerable improvement in the computational efficiency.

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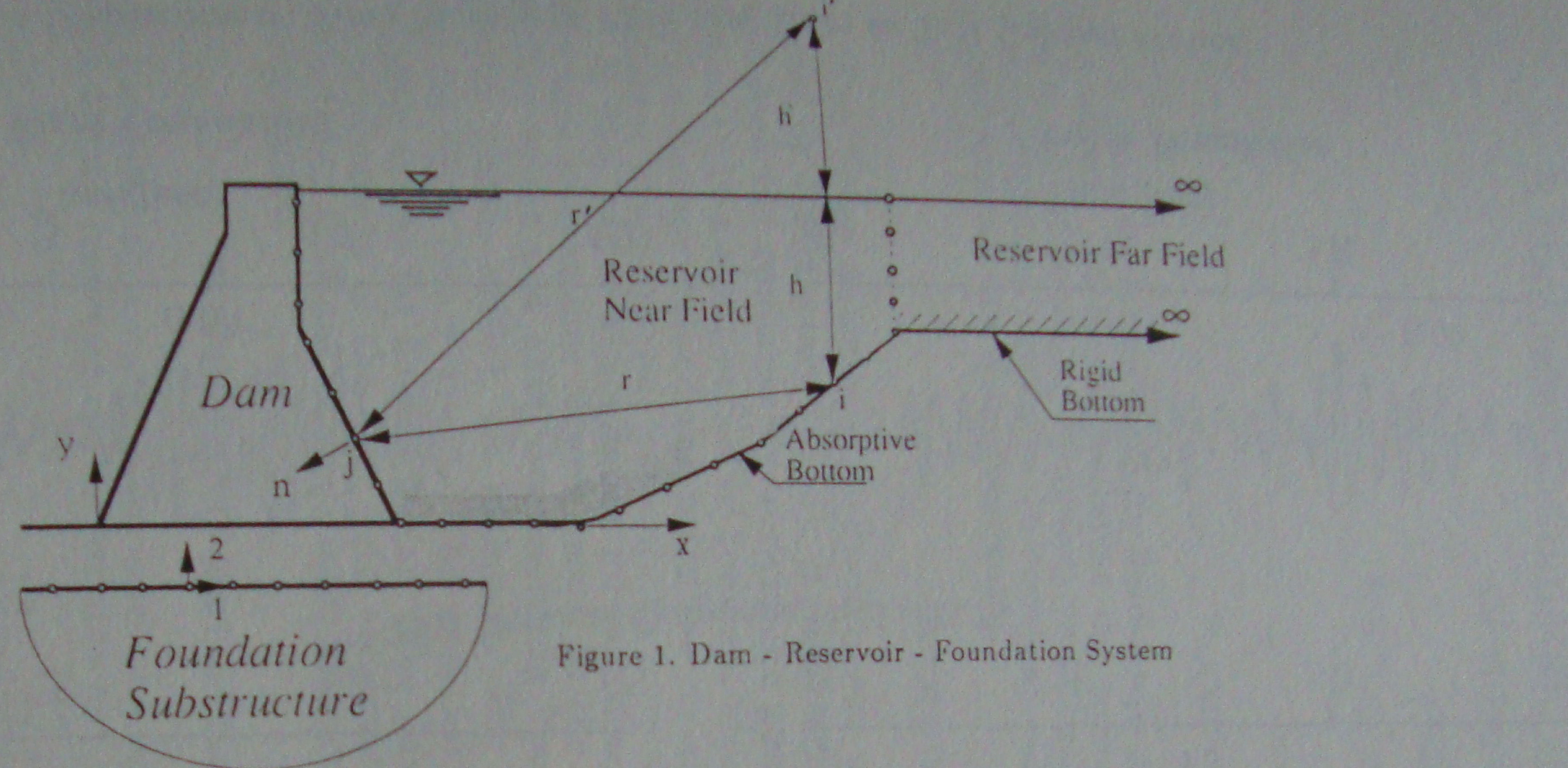

Figure 1. Dam - Reservoir - Foundation System

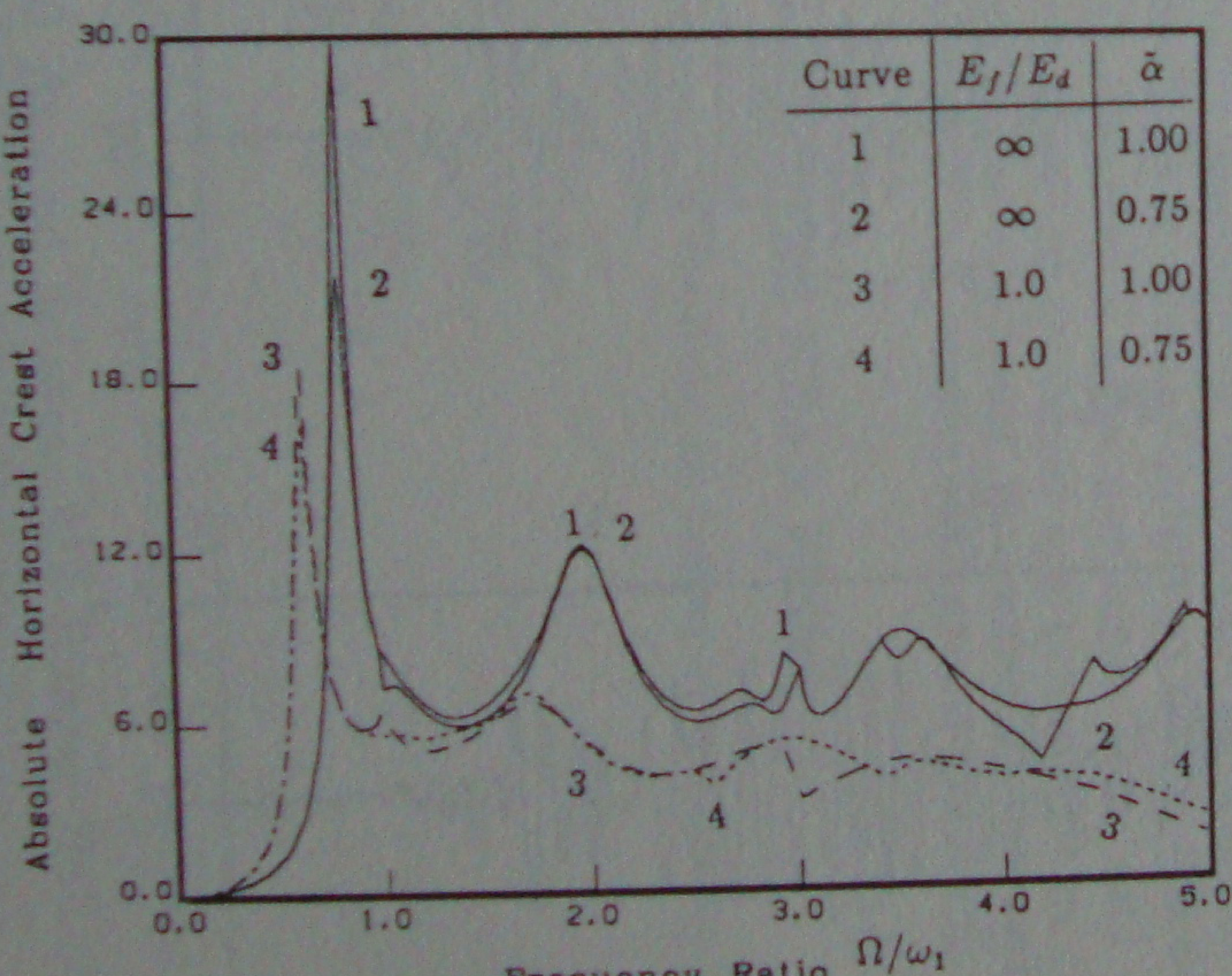

Figure 2 Response Function to Horizontal Excitation

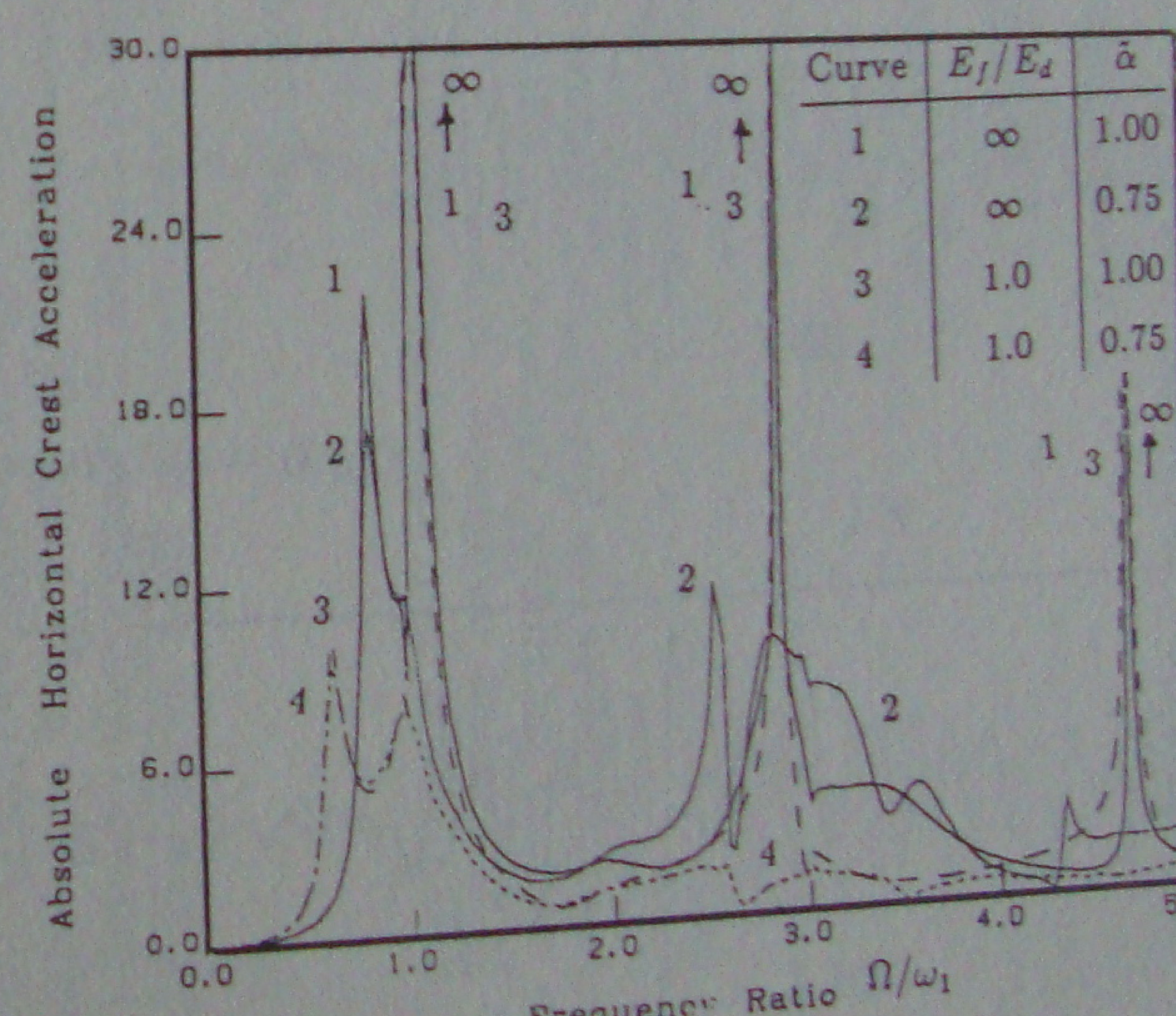

Figure 3 Response Function to Vertical Excitation

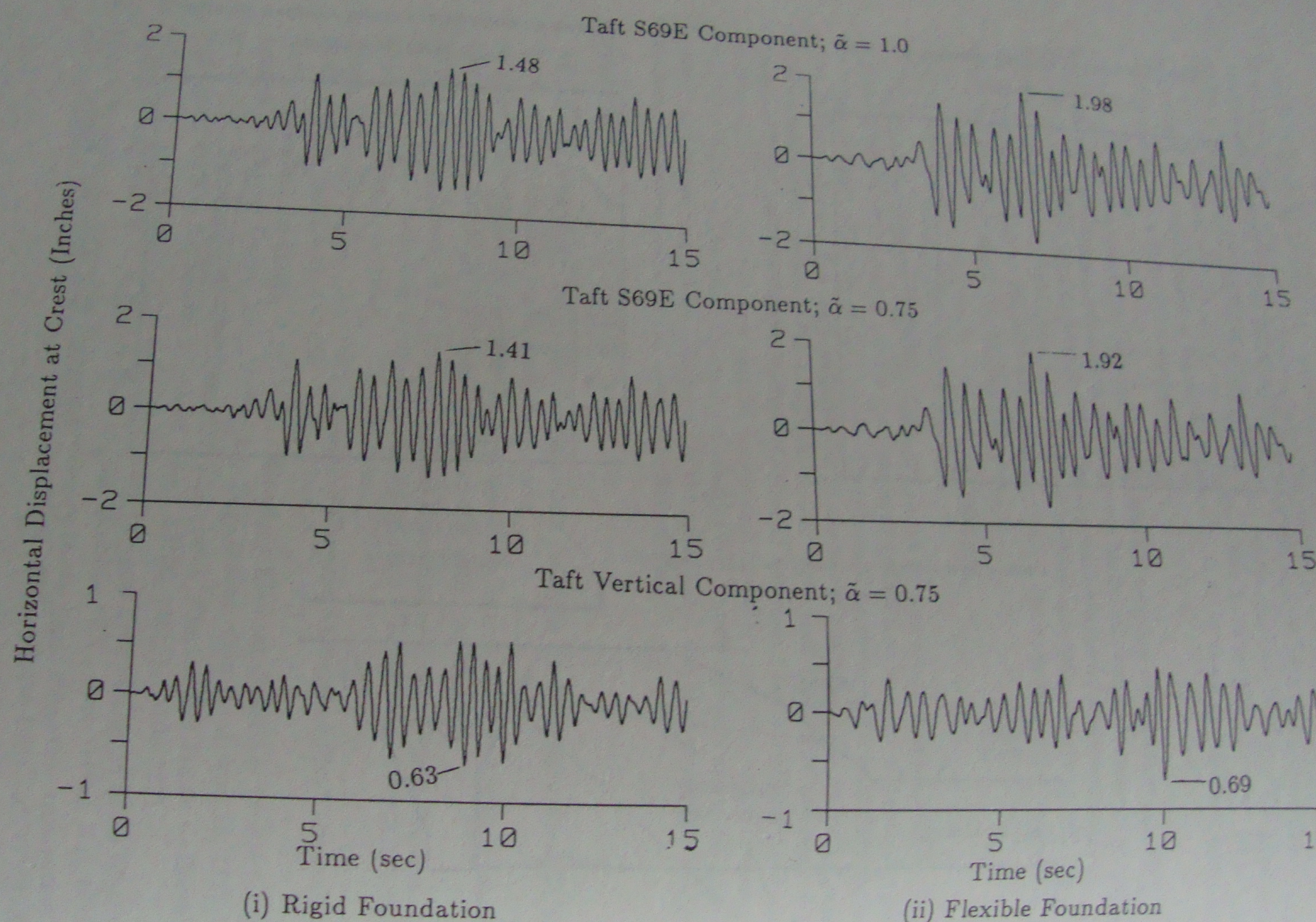

Figure 4 Displacement Time History of Pine Flat Dam to Taft Ground Motion